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NAVAL POSTGRADUATE SCHOOL
Monterey, California

DESIGN BUCKLING CURVES

FOR

GUN LAUNCHED ROCKETS

BY

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ABSTRACT:

An investigation of elastic buckling of gun-launched motor cases is considered. The SATANS finite difference computer program was used for the analysis. Comparative results from other buckling models were obtained to verify the results. The results are summarized in a set of design curves and show that for the proposed design parameters, the elastic behavior is exceeded before buckling can occur.

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TABLE OF CONTENTS

NOTATION

a, a_m	acceleration, maximum acceleration (in g's)
E	Young's modulus of elasticity
G	Shear modulus
h	thickness of motor case
L	length of motor case
N	buckling load per unit circumference
p_i	radial pressure due to slumping propellant
p_o	escaping breech force pressure
p_s	motor case body force due to acceleration
P_a	aft edge force
P_f	forward edge force
R	radius of motor case
W_f	weight of structure fore to motor case
ν	Poisson's ratio
σ	axial stress
σ_y	yield stress
ω	density

INTRODUCTION

This report is concerned with the elastic buckling of gun-launched motor case structures which are subjected to large axial g loading during launch. Acceleration loading up to 10,000 g is typical. The axial loading is accompanied by a lateral loading due to an internal slumping propellant which also provides additional stiffness. In a previous report (ref. 1, Appendix D) a computation of the buckling load for a motor case with 0.08 inch thickness, 1.5 inch radius, and 9.5 inch length showed buckling occurs at a stress of 356 ksi, exceeding the elastic limit of the high performance heat-treated steel (200-250 ksi). Although that analysis indicated elastic buckling was not probable, a parametric study of the elastic buckling of the case for various values of radius, length, thickness and forward mass was undertaken in the event steels with higher heat treatment were used. A finite difference computer program, known as SATANS (ref. 2) and capable of performing geometrically nonlinear static and dynamic analyses of arbitrarily loaded shells of revolution, was used to compute the static buckling loads.

DESCRIPTION OF THE MOTOR CASE

The motor case structure is a cylindrical shell of 4130 heat-treated steel alloy. From MIL-HDBK-5A, February 8, 1966:

Elastic modulus in tension and compression, $E = 30\times10^{6}$ psi

Modulus of rigidity, $G = 11\times10^{6}$ psi

Poisson's ratio $\nu = 0.32$

Density, $\omega = 0.283\#/in^{3}$

The design parameters furnished by The Naval Weapons Center, China Lake, are:

radius, $R = 2.5$ inches

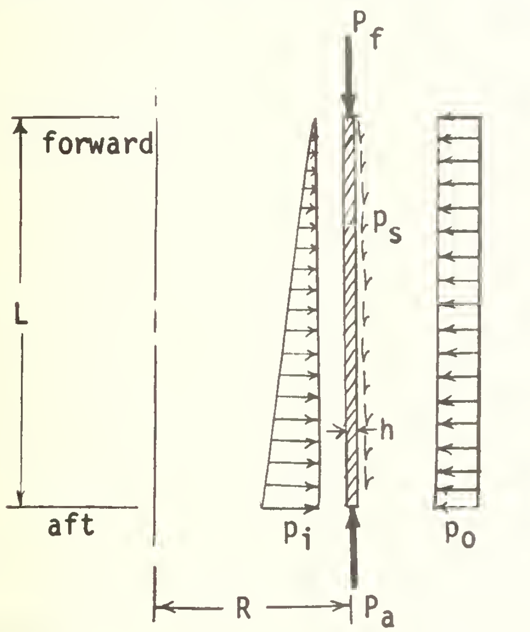
length, L, between 19 and 24 inches

thickness, h, between 0.125 and 0.250 inches

forward structure weight, W_f, between 45 and 70 lbs.

DESCRIPTION OF THE EXTERNAL LOAD

The external load on the motor case structure was originally idealized as shown in the accompanying figure.



P_f is the force per unit circumference along the forward edge and is associated with the acceleration of the forward structure.

$$P_f = \frac{W_f}{2\pi R} \cdot a_m$$

where a_m is the maximum acceleration in g's.

p_s is the axial distributed force per unit circumference due to the acceleration of the motor case itself,

$$p_s = \omega h a_m$$

where ω is the case material density.

P_a is the aft force per unit circumference obtained by equilibrium considerations

$$P_a = P_f + p_s . L$$

Finally, p_i and p_o are the lateral pressures associated with the slumping propellant and escaping breech pressure, respectively. These two pressures were neglected in the analysis. The breech pressure p_o can be minimized by good obdurator design, and it would be difficult to determine the magnitude under any circumstances. The effect of the

slumping propellant pressure p_i was examined in detail and is the subject of another report. It was determined that the maximum internal pressure would be of the order of 10 ksi for a motor case with a 2.5 inch radius, a 10,000 g acceleration, and a 20 inch length. Theory predicts and preliminary analyses verified that the internal pressure stiffens the cylinder. This fact, together with the uncertainty of the propellant parameters, provided the basis for neglecting the internal pressure in the analyses.

ANALYSIS

The validity of the results, which are given in the next section, is considered here. The boundary conditions on both ends are taken as simple supports.

The various thin shell theories are associated most often with slightly different approximations and assumptions. For example, Love, Donnell, Reissner, Flüggë, Sanders and others have thin shell theories associated with the particular thin shell models they have proposed. The finite difference program SATANS used in this analysis is based on the Sanders model for thin shells (ref. 2), which includes the nonlinear terms in the strain-displacement and equilibrium relations. The results obtained from SATANS for a cylinder subjected to an axial load are compared to those from three other sources; the Flüggë equations (ref. 3), the classical (linear) buckling equation (ref. 4), and empirical results (ref. 5). The results for the critical buckling load N and corresponding stress σ are given in Table 1 for various values of R/h. In all cases, Young's modulus and Poisson's ratio are 30×10^6 psi and 0.32, respectively.

Table 1. N is in lb./in. σ is in psi.

	SATANS	FLUGGË	CLASSICAL	EMPIRICAL
R/h = 288 L = 40", R = 2"	N = 360 σ = 52,700	N = 315 σ = 45,500	N = 439 σ = 64,300	N = 144 σ = 20,800
R/h = 144 L = 40", R = 2"	N = 1062 σ = 76,600	N = 1112 σ = 80,400	N = 1755 σ = 126,800	N = 633 σ = 45,700
R/h = 92 L = 40", R = 2"	N = 3264 σ = 149,000	N = 3000 σ = 137,000	N = 4386 σ = 200,000	N = 1656 σ = 75,600
R/h = 16 L = 40", R = 2"	N = 46,300 σ = 370,000	N = 43,300 σ = 346,000	Not Applicable	N* = 58,500 σ* = 468,000

In all cases the results given in Table 1 show the classical (linear) buckling theory gives buckling loads which are not in agreement with any of the other sources. Its inclusion here was simply to demonstrate this shortcoming. It is also noted that for R/h = 16 the classical theory cannot be used.

The first three cases of the table show that for very thin shells, say R/h > 50, the SATANS and Fluggë theories agree quite well but give buckling loads twice as large as the empirical results. This difference has been noted by many investigators (see ref. 5, page 509), and is accounted for by shell imperfections and load eccentricity. For the motor case dimensions provided by NWC, the R/h ratios lie between 10 and 20.

Here Fluggë and SATANS agree quite well and if the empirical results are extrapolated, additional agreement is obtained. In addition to this corroboration between theory and empirical results, it is noted that for these shells with $R/h \approx 15$, the theoretical solution is on the safe side of the extrapolated empirical result. The superscript asterisks on the empirical N and σ for $R/h = 16$ indicates these results have not been achieved in the laboratory, rather they have been obtained by extrapolation to provide some comparison here.

DESIGN BUCKLING CURVES

The results obtained from SATANS for the range of design parameters given by NWC are shown in figures 1 through 5. Figures 1 through 3 are plots of acceleration versus L/R for the minimum and maximum h/R ratios (0.05 and 0.1 respectively). Each figure is for a specified forward weight, W_f, of 45, 60, and 75 pounds respectively. The buckling curves are specifically labeled. The remaining curves denote failure by yielding for 200, 250, and 300 ksi steels. The most important observation here is the one anticipated by the previous analysis; elastic buckling will not occur. If buckling is to occur, it must be plastic.

The small variation of acceleration with L/R ratio suggests the alternate plots of figures 4 and 5 which show acceleration versus W_f for minimum and maximum h/R ratios for fixed L/R ratios.

DESIGN TECHNIQUE

Figures 1 through 3 or figures 4 and 5 may be used for design by selecting any three out of the four quantities, a, h/R, L/R, and W_f, and obtaining the remaining fourth quantity from the lower of the two curves associated with buckling and elastic yield. In all cases considered

here the latter is the failure mode.

The values of h/R equal to 0.05 and 0.10 are the minimum and maximum values of h/R obtained from the NWC design parameters, R = 2.5 inches and h between 0.125 and 0.250 inches. The buckling curve for h/R equal to 0.075 was determined to provide a means of interpolation for values of h/R intermediate to 0.05 and 0.10. The proximity of the 0.075 curve to the mean of the 0.05 and 0.10 curves suggests that linear interpolation may be used. As an example consider the case for h/R = 0.625, L/R = 7.6 and W_f = 60 pounds. From figure 4 we find a_m equal to 17,600 g and 42,300 g for h/R equal to 0.05 and 0.10 respectively. By linear interpolation the maximum acceleration for static buckling when h/R = 0.0625, L/R = 7.6, and W_f = 60 pounds is

$$a_m = \frac{.625 - .50}{.75 - .50} \quad (17{,}600 + 42{,}800) = 35{,}200 \text{ g.}$$

Because figures 1 and 5 have log scale ordinates, direct scaling of the curves is not valid.

The relation governing the initial yield curves is

$$\sigma = \frac{F}{A} = \frac{ma}{A} = \frac{(W_f + W_s)a}{2\pi Rh}$$

where W_s is the weight of the motor case, W_f, is the weight of the fore structure, and the acceleration a is in g units. Solving for the acceleration when the stress is at yield gives

$$a_m = \frac{2\pi Rh}{W_f + W_s} \sigma_y = \frac{2\pi Rh}{W_f + 2\pi Rh \,.\, \omega} \sigma_y$$

This equation may be solved for any values of the design parameters. It may be noted that among all variables the maximum acceleration, a_m, is linearly proportional to yield stress, σ_y, only.

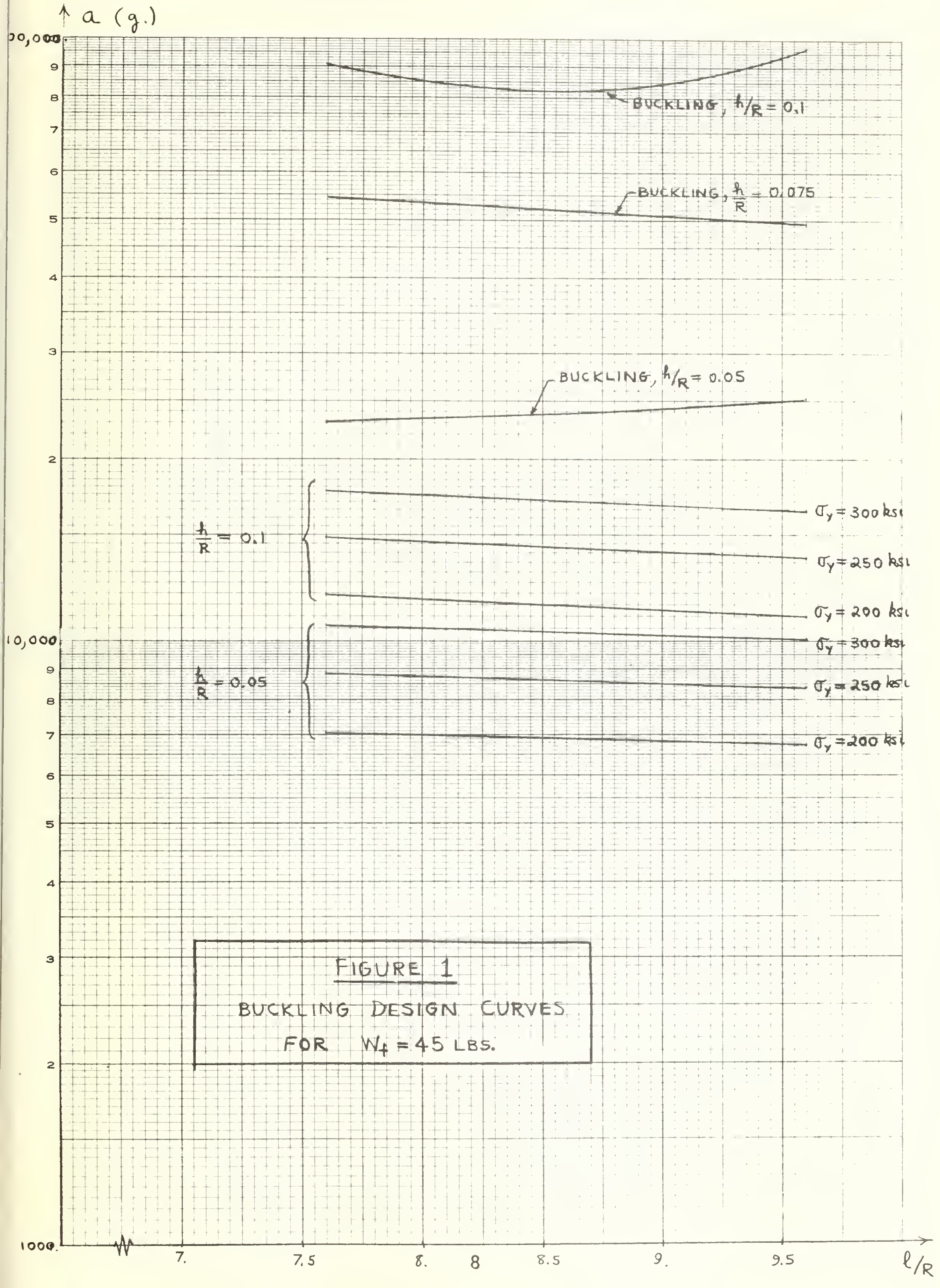


FIGURE 1

BUCKLING DESIGN CURVES FOR $W_f = 45$ LBS.

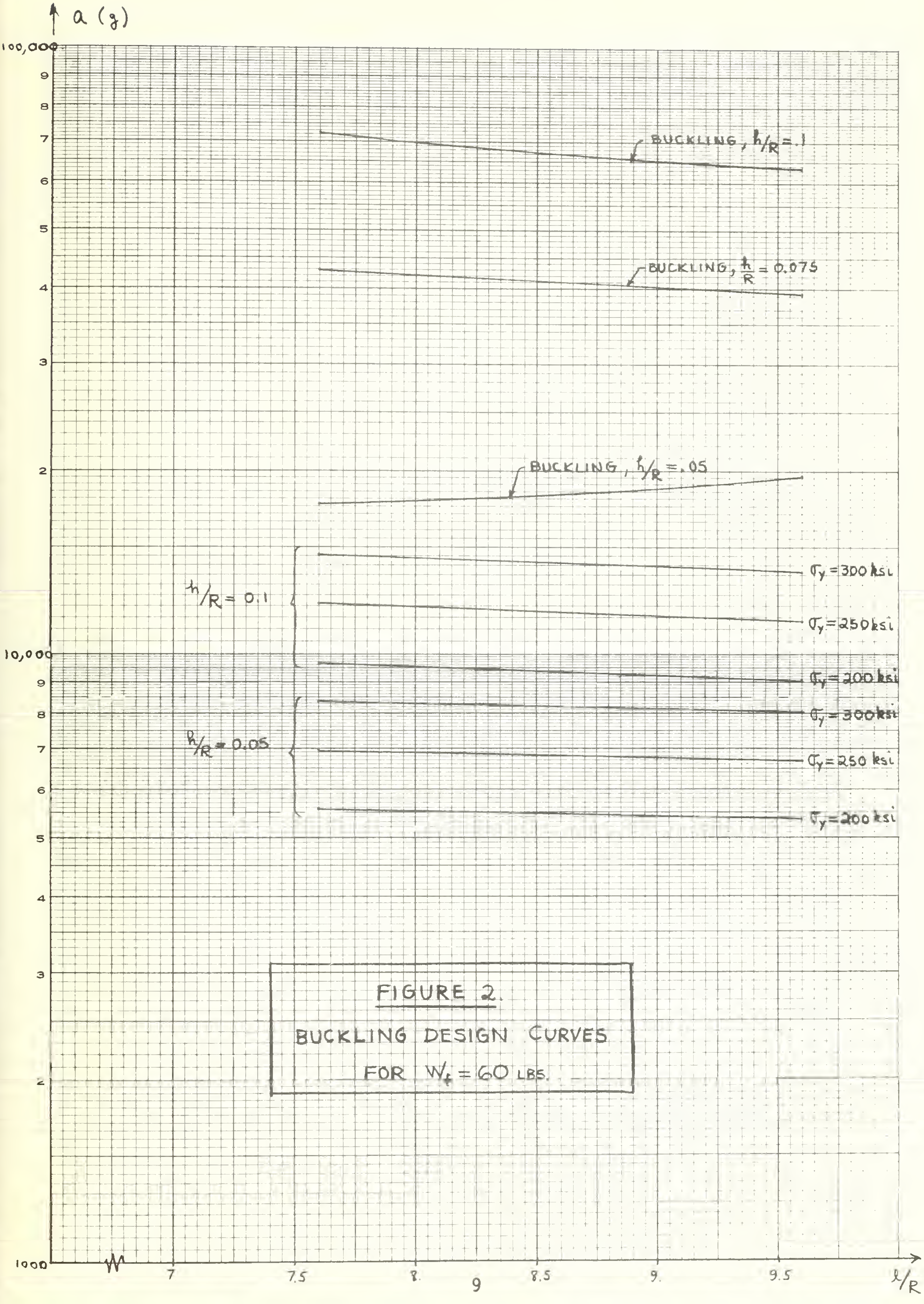


FIGURE 2.

BUCKLING DESIGN CURVES

FOR W_t = 60 LBS.

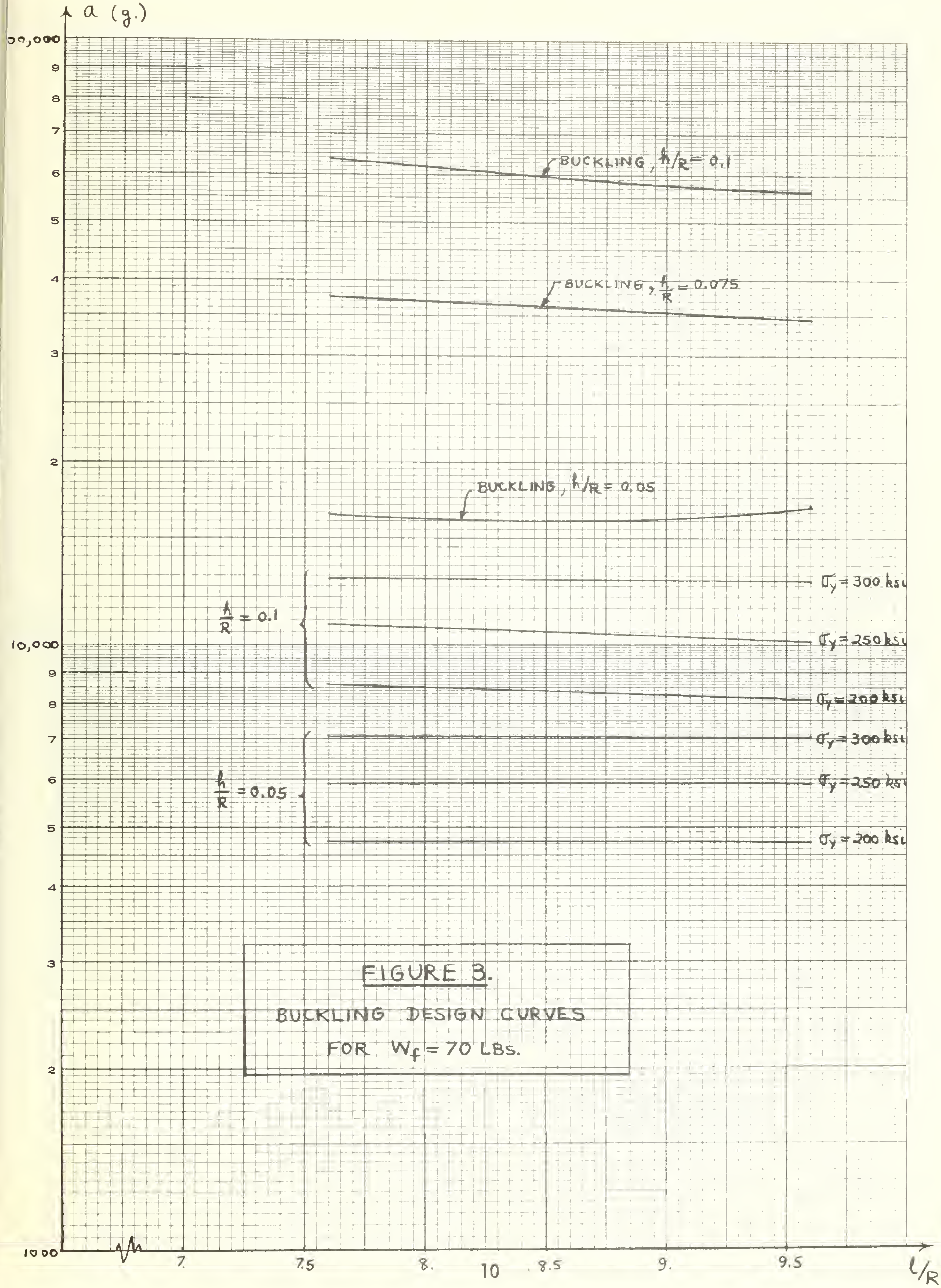


FIGURE 3.

BUCKLING DESIGN CURVES FOR $W_f = 70$ LBS.

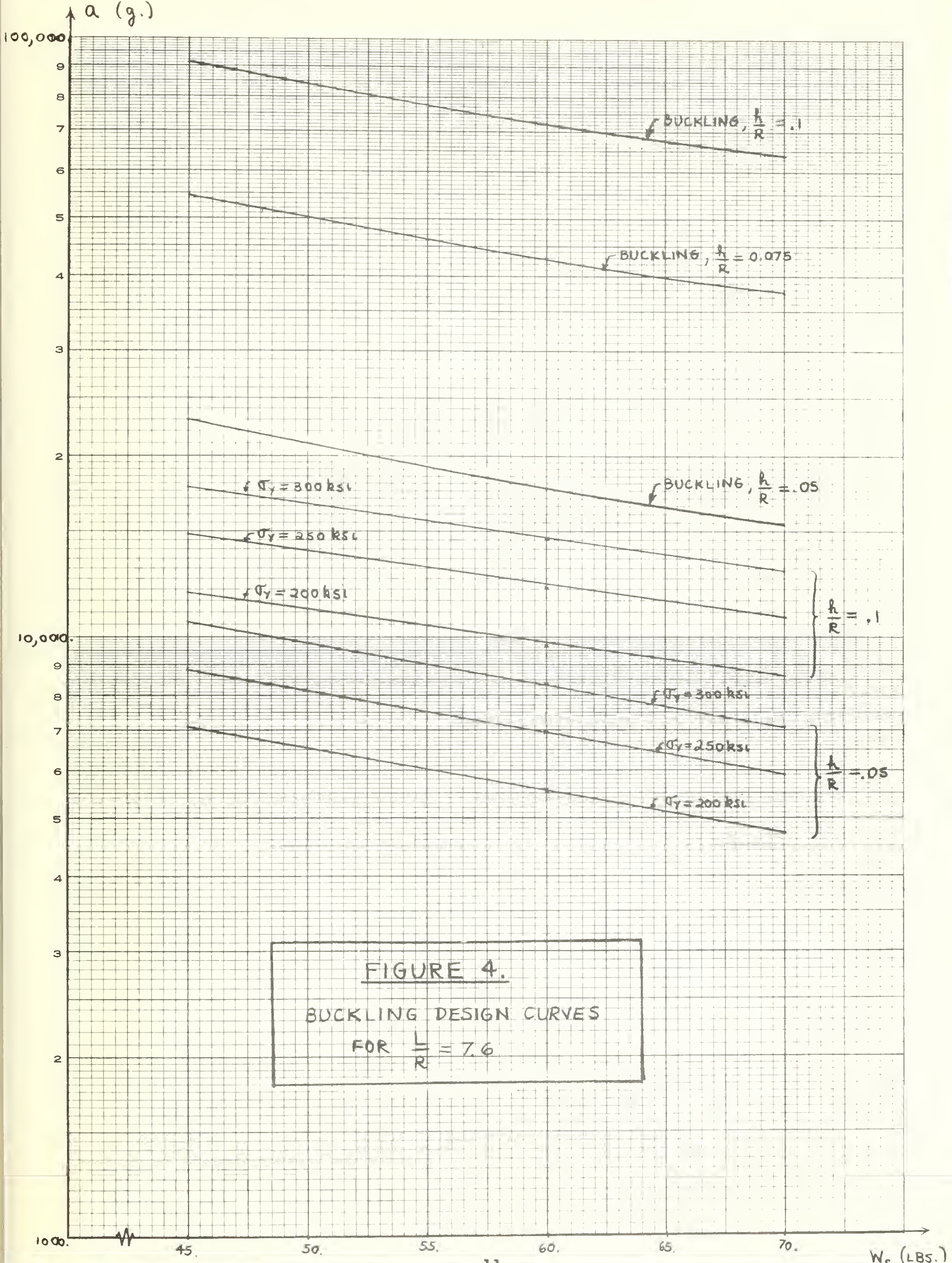


FIGURE 4.

BUCKLING DESIGN CURVES FOR $\frac{L}{R} = 7.6$

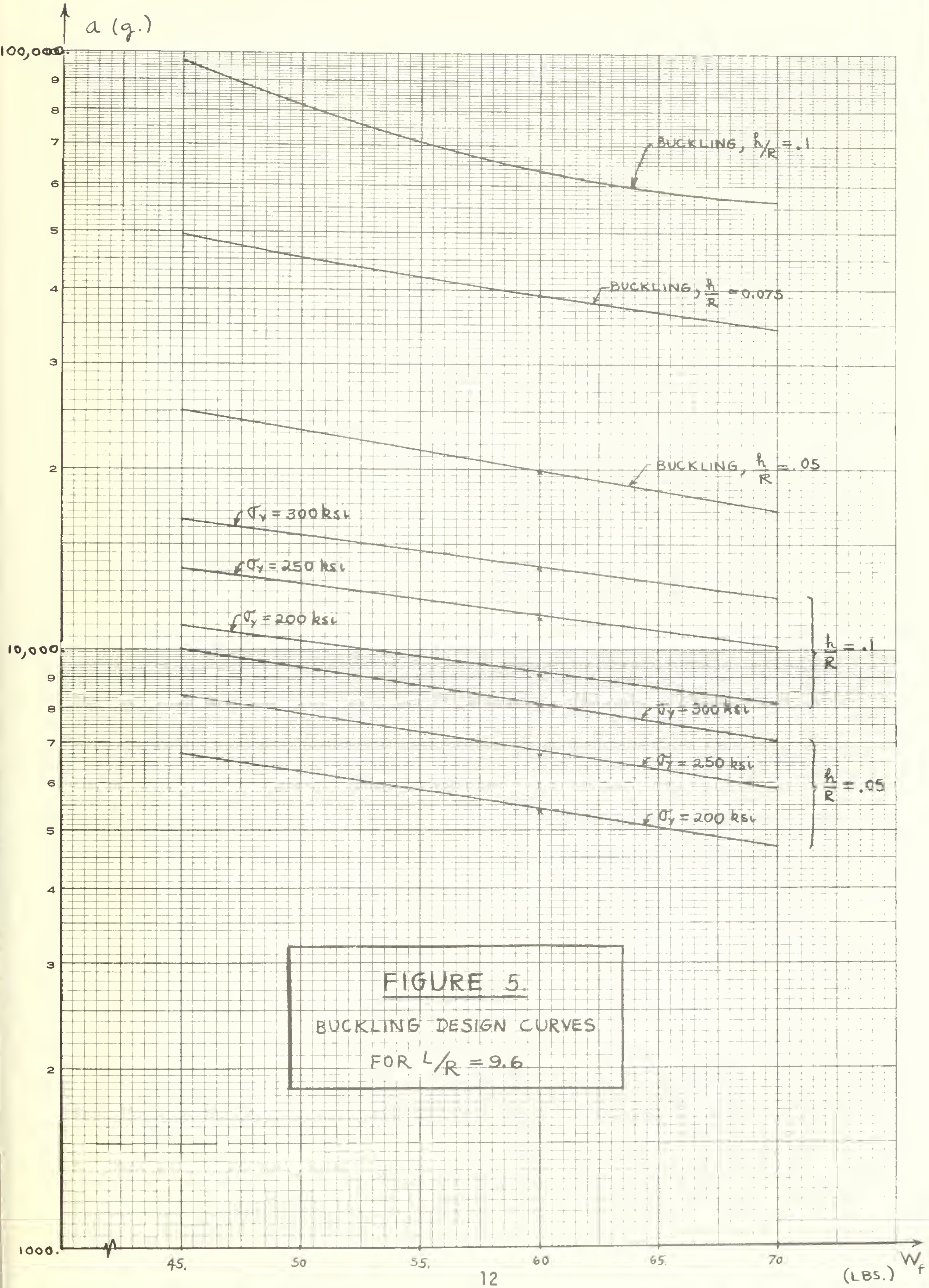


FIGURE 5.

BUCKLING DESIGN CURVES

FOR $L/R = 9.6$

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